

THE COHERENCE OF PRIMORDIAL FLUCTUATIONS PRODUCED DURING INFLATION

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Abstract

The behaviour of quantum metric perturbations produced during inflation is considered at the stage after the second Hubble radius crossing. It is shown that the classical correlation between amplitude and momentum of a perturbation mode, previously shown to emerge in the course of an effective quantum-to-classical transition, is maintained for a sufficiently long time, and we present the explicit form in which it takes place using the Wigner function. We further show with a simple diffraction experiment that quantum interference, non-expressible in terms of a classical stochastic description of the perturbations, is essentially suppressed. Rescattering of the perturbations leads to a comparatively slow decay of this correlation and to a complete stochastization of the system.

1 Introduction

According to the inflationary scenario, all inhomogeneities in the Universe were produced quantum mechanically from quantum vacuum fluctuations of inflaton field(s) (effective scalar field(s) driving inflation) and gravitational field. Thus they are of genuine quantum gravitational origin and still observable today. The simplest variant predicts an approximately flat, or Harrison-Zeldovich, initial power spectrum for both scalar (density) perturbations [1] and gravitational waves (GW) [2]. The corresponding fluctuations $\frac{\Delta T}{T}(\theta, \varphi)$ of the cosmic microwave background (CMB) were detected by the COBE large-angle, and a number of medium-angle, experiments. Clearly, all inhomogeneities measured presently appear to us as classical. So, a question of utmost importance is how and when does this effective quantum-to-classical transition occur?

The amplitude of the observed fluctuations for a mode \mathbf{k} is such that the condition $\langle n_{\mathbf{k}} \rangle \gg 1$, where $n_{\mathbf{k}}$ is the particle number operator, is satisfied by a large margin (see [3, 4] for exact conditions). However, this condition is not sufficient by itself to determine the nature of the quantum-to-classical transition, more information about the quantum state of each mode is needed. In particular, this transition is completely different for a coherent and a strongly squeezed state. The free non-relativistic particle at very late times provides yet another illustration of this dramatic difference [5]. In the first case, the quantum mode can be approximated by a *deterministic* classical time- and space-dependent field (a Bose condensate). But the generation of fluctuations at the inflationary stage through particle creation out of the vacuum (described by a Bogolyubov transformation in the Heisenberg representation) leads to a squeezed state. As emphasized recently [3], in the limit of extreme squeezing, a quantum mode can be approximated by a classical stochastic field with *random* (Gaussianly distributed) amplitude and *fixed* phase. This fundamental result can be extended to non-vacuum states [4], with non-Gaussian distributed amplitude. In this quantum to classical transition, it is the quantum correlation between the growing and decaying parts of the perturbations that gets lost (the decaying part may even be completely damped, e.g. by viscosity). As no interaction with the environment is needed here, in contrast to usual decoherence [6], it was called “decoherence without decoherence” in [3]. The sensitivity of this process to a type of interaction with the environment was considered in [7, 5]: when the decaying mode is negligible, as is the case in the high squeezing limit, the field amplitude basis is the classical “pointer” basis for interactions that are in field amplitude space. As this is the case for most interactions, the loss of quantum coherence between the growing and decaying parts of the perturbations is essentially independent of the details of these interactions.

These previous studies were mainly dealing with the behaviour of the perturbations during the inflationary stage or after it, but still before the second Hubble radius crossing, i.e. for $\lambda \equiv \frac{a(t)}{k} \geq R_H$, where $a(t)$ is the scale factor of the FRW metric while $R_H = \frac{c}{\dot{a}}$, $c = 1$ (see however eqs 54-56 and the discussion on p.388-389 in [3]). Now we concentrate on the late stage of their evolution

when $\lambda \ll R_H$. The fixed phase of a perturbation mode mentioned above means that this part of the quantum coherence is *not* destroyed by decoherence up to the moment of the second Hubble radius crossing. It reveals itself in the form of a *classical* correlation between amplitude and momentum of the mode and is all that remains from the initial quantum coherence. Using the Wigner function, we show in Sec.2 that this correlation is maintained even for $\lambda \ll R_H$. Then, in Sec.3, we consider a simple diffraction experiment which shows again that quantum interference, non-expressible in terms of a classical stochastic description of the perturbations, is essentially suppressed. This long-existing classical correlation of cosmological perturbations was overlooked in many papers which discuss their decoherence and entropy (see e.g. [8, 9]). For scalar perturbations, this correlation leads to Sakharov oscillations in the matter transfer function and to oscillations in the dispersion values C_l of the $\frac{\Delta T}{T}$ multipoles. Note, however, that these oscillations are of purely dynamical origin and do *not* discriminate between quantum and classical origin of the perturbations (see also the discussion in [10]). Of course, this correlation will not remain forever. Rescattering of perturbations and other processes as well finally lead to a complete loss of the initial quantum correlations, i.e. to a complete decoherence. As discussed in Sec.4, the characteristic time for the latter process depends crucially on the type and wavelength of the perturbations under consideration.

2 Classical correlation

The time evolution of the perturbations in the regime $\lambda \gg R_H$ leads to an extreme squeezing which remains when λ becomes smaller than R_H (see e.g. [3]). As a result, the quantum coherence is expressible in classical stochastic terms: for a given “realization” $y_{\mathbf{k}}$ of the fluctuation field, we have for its canonical momentum $p_{\mathbf{k}} \simeq \frac{g_{k2}}{f_{k1}} y_{\mathbf{k}} \equiv p_{\mathbf{k},cl}$, the *classical* momentum for large squeezing, i.e., for $|r_k| \rightarrow \infty$. Here, f_k , resp. g_k , is the amplitude, resp. momentum, field mode in the Heisenberg representation (we follow the conventions of [3]), and the index 1 (2) refers to its real (imaginary) part, the same convention is used for all complex quantities. The conformal time $\eta \equiv \int^t \frac{dt'}{a(t')}$ is used in the following. This almost perfect classical correlation is best seen with the help of the Wigner function, and we show now what precise form it takes today for these perturbations deep *inside* the Hubble radius. Their modes f_k can be written as

$$f_k = D_1 \sin(k\eta + \xi_k) + iD_2 \cos(k\eta + \xi_k) \quad (1)$$

where D_1, D_2 are real, and ξ_k is some phase. Extreme squeezing, or equivalently the almost complete disappearance of the decaying mode during the evolution outside the Hubble radius, manifests itself in the ratio $\frac{D_2}{D_1} \propto \exp(-2r_k)$. This is of the order 10^{-100} or less for the largest cosmological scales! For an initial vacuum state the wave function is and remains Gaussian, while the Wigner function W is positive definite. Then, for modes deep inside the Hubble radius, given by (1),

the following Wigner function is obtained (we consider half of the phase-space)

$$W(y_0, p_0; \eta) = \frac{1}{\pi} e^{-\frac{y'^2}{D_1^2}} e^{-\frac{(p'/k)^2}{D_2^2}}, \quad \tan(k\eta + \xi_k) \gg e^{-2r_k}, \quad (2)$$

where the (y', p') -frame is rotating in the (y_0, p_0) -plane while the rotation is clockwise. The y' axis makes an angle φ_k with the y_0 axis that is given by $\varphi_k = \frac{\pi}{2} - k\eta - \xi_k$, the latter is just the squeezing angle. The rotation velocity $\omega \equiv \frac{d\varphi_k}{dt}$, where t is the cosmological time, is given by $\omega = 2\pi \lambda_{phys}^{-1}$, with $\lambda_{phys} \equiv a \frac{2\pi}{k}$. Taking into account that $D_1 \gg D_2$, it is clear that the Wigner function (2) is concentrated along the y' axis. This just corresponds to the classical random process $y = D_1 \sin(k\eta + \xi_k) e_y$, where e_y is classical Gaussian random variable with unit variance. The typical volume is a (rotating) highly elongated ellipse whose thickness is tremendously small and proportional to the amplitude of the decaying mode. It is in this in practice unobservable thickness, the variance of the quantity $p - p_{cl}$ when the ellipse is in horizontal position, that the quantum coherence not expressible in classical stochastic terms is “hidden”. We note that this typical volume remains constant during all the time evolution of the fluctuations inside as well as outside the Hubble radius [11]. It is clear that one is not allowed to average over the angle φ_k , as this would not reflect the remaining quantum coherence and the resulting fixed phase of the perturbations. For the largest cosmological scales it would correspond to averaging over times of the order of the age of the universe! Both terms in (1) will be of the same amplitude during a tiny time interval δt , per (half) oscillation, when they are both of order $\sim D_1 e^{-2r_k}$. We have $\delta t \sim 10^{-80}$ sec or less for wavelengths on cosmological scales $\sim 100 h^{-1} \text{Mpc}$! Note that typical times for the loss of quantum coherence between these terms due to interaction with other fields are even less than δt [5]. During this short time interval one has

$$W(y_0, p_0) \simeq \frac{1}{\pi} e^{-\frac{(p_0/k)^2}{D_1^2}} e^{-\frac{y_0^2}{D_2^2}}, \quad (3)$$

which still corresponds to the classical correlation between amplitude and momentum. In conclusion, we still have zero measure “trajectories” in phase space satisfying $p = p_{cl}(y)$ with an accuracy well beyond observational capabilities.

3 A diffraction experiment

We shall now show with a concrete diffraction (gedanken) experiment that quantum interferences are quasi-classical up to an accuracy well beyond observational capabilities. We consider the following experiment: at time η_1 , an apparatus rejects all fluctuations with amplitudes outside the range $[-\Delta, \Delta]$. The fluctuations that got through are then observed at some later time η_2 . Our experiment corresponds physically to a fixed “slit” of size 2δ in ϕ -space and to a slit of time-dependent size $2a\delta \equiv 2\Delta$ in y -space where $a(\eta)$ is the scale factor. For our sake it

is sufficient to consider the real part, $y_{\mathbf{k}1}$ and we further introduce the simplified notation $y_1(\eta_1) \equiv x_1$, $y_1(\eta_2) \equiv x_2$.

We would like to show that the observed ‘‘pattern’’ corresponds to the predictions of a classical stochastic process to tremendous high accuracy. The outcome of our experiment is encoded in the quantity $I(\Delta)$, or better $|I(\Delta)|^2$ which gives the probability distribution at time η_2

$$I(\Delta) \equiv \int_{-\Delta}^{\Delta} dx_1 K(x_2, \eta_2; x_1, \eta_1) \Psi(x_1, \eta_1) , \quad (4)$$

where the propagator $K(x_2, \eta_2; x_1, \eta_1)$ gives the probability amplitude to go from x_1 at time η_1 to x_2 at time η_2 [14]. We perform our diffraction experiment with fluctuations on cosmological scales deep inside the Hubble radius, $\frac{k}{a} \gg H$. Hence the wave function $\Psi(x_1, \eta_1)$ in (4) corresponds to a highly WKB state with $\Psi(x_1, \eta_1) = R(x_1, \eta_1) e^{\frac{i}{\hbar} S_{cl}(x_1, \eta_1)}$ [3, 5]. After some calculation the following result is obtained

$$K(x_2, \eta_2; x_1, \eta_1) \Psi(x_1, \eta_1) \propto R(x_1, \eta_1) e^{\frac{i}{\hbar} \frac{k}{\sin k \Delta \eta} \left(\frac{f_1(\eta_2)}{f_1(\eta_1)} x_1^2 - 2x_2 x_1 \right)} . \quad (5)$$

There is a range of extremely narrow slits, of size 2δ in ϕ -space or equivalently 2Δ in y -space, for which the real part of the wave function can be taken to be constant across the slit, while the complex phase cannot, and we assume our slit satisfies this condition. Hence the details of the wave function are irrelevant provided the state is of the WKB type. Therefore, $I(\Delta)$, see (4), is given by an expression of the type ([12], 2.549.3-4)

$$I(\Delta) \propto \sqrt{\frac{\pi}{2d}} e^{-i \frac{b^2}{d}} \left\{ C\left(\frac{d\Delta + b}{\sqrt{d}}\right) + C\left(\frac{d\Delta - b}{\sqrt{d}}\right) + i[S\left(\frac{d\Delta + b}{\sqrt{d}}\right) + S\left(\frac{d\Delta - b}{\sqrt{d}}\right)] \right\} , \quad (6)$$

where $d \equiv \frac{k}{\hbar \sin k \Delta \eta} \frac{f_1(\eta_2)}{f_1(\eta_1)}$, $b \equiv -\frac{k}{\hbar \sin k \Delta \eta} x_2$. The result is thus expressed in terms of the Fresnel integrals $C(x)$, $S(x)$. Even a slit of very small constant size 2δ in ϕ -space will eventually satisfy $2\Delta \gg 1$ at time η_1 due to the inflationary growth of the scale factor. An essential property here of the Fresnel integrals is that they both tend to $\frac{1}{2}$ for $x \rightarrow \infty$, they are close to this asymptotic value already at $x \simeq 10$ (see e.g. [13]). Hence, by inspection of (6), it is seen that for $|d| \Delta \gg 1$ the expression (6) rapidly becomes zero when $|b| > |d|\Delta$, within a range (in y -space) which is exceedingly small compared to $|d| \Delta \gg 1$. This means that we get a sharp pattern with $|\Psi(x_2)|^2 \simeq |\Psi(0)|^2$ in the range

$$-\frac{f_1(\eta_2)}{f_1(\eta_1)} \Delta \leq x_2 \leq \frac{f_1(\eta_2)}{f_1(\eta_1)} \Delta . \quad (7)$$

This is precisely the result one would expect with a classical stochastic process.

4 Loss of remaining coherence

As we have seen, there exists a long period in the evolution of a perturbation after the second Hubble radius crossing when it oscillates with a fixed phase and occupies a very small volume in phase-space. Certainly, this cannot proceed forever, and eventually we may expect total stochastization and total loss of the initial coherence (correlation) to take place. How fast it occurs, if it occurs at all, depends on the concrete system under consideration.

a) Gravitational waves: The first process which can lead to stochastization of the phase is graviton-graviton scattering. As was discussed in [7], the pointer observable (the amplitude in this case) does not commute with the corresponding interaction Hamiltonian. However, a simple estimate shows that this interaction is ineffective even for wavelengths crossing the Hubble radius for the first time towards the end of inflation. Another process seems to be more effective: generation of a secondary GW background by matter after the second Hubble radius crossing. This process does not respect the phase of the primordial background and produces GW with a stochastic, uniformly distributed, phase. To screen the fixed phase of the primordial background, the spectral density $\frac{d\epsilon_g}{d\ln\nu}$ of the secondary background should be larger than the primordial one which is $< 10^{-14}\rho_{crit}$, with ρ_{crit} being the critical energy density. While there exist astrophysical sources which may produce such a large secondary background with frequencies $\nu > 10^{-4}\text{Hz}$, a large secondary background at smaller frequencies requires exotic sources in the early Universe like, for example, cosmic strings (see e.g. [15]).

b) Scalar perturbations: Scalar perturbations with present scale $\lambda > R_{eq} \sim 15 h^{-2} \text{ Mpc}$ crossed the Hubble radius last during the matter-dominated stage. In contrast to GW, they do not oscillate. For them, the classical correlation between amplitude and momentum discussed above simply means that (in the linear approximation) the velocity potential Ψ_{vel} is proportional to the gravitational potential: $\Psi_{vel} = \Phi t$ (in other words, velocity and acceleration are parallel). Non-linear effects change the relation between both potentials without destroying it, until shell-crossing occurs. After that, the motion of matter becomes multistreamed and cannot be described anymore with one-fluid hydrodynamics. This results in the growth of the phase-space occupied by the perturbations. It is natural to expect that the initial phase relation will be destroyed in the high density regions with $\frac{\delta\rho}{\rho} \gg 1$ where gravitational relaxation took place. Hence, the characteristic time τ for the loss of the remaining coherence is $\tau \sim t_k \Phi^{-\frac{3}{2}}$, where $\Phi \equiv \sqrt{k^3 \langle \Phi_k^2 \rangle} \sim 10^{-5}$ is the r.m.s. amplitude of the initial gravitational potential and t_k is the second Hubble radius crossing time.

Scalar perturbations with present scale $\lambda < 15h^{-2} \text{ Mpc}$ passed through the stage of acoustic oscillations in the past. At that stage, Thompson scattering of photons by electrons leads to a dissipative process, the so-called Silk damping.

However, it is not clear yet if this process destroys the phase correlation quicker than it damps the acoustic oscillations themselves. In particular, though numerical simulations clearly show that both the multipoles C_l and their acoustic peaks are strongly damped for $l > 1000$, it remains yet to be shown that the latter is damped quicker. Hence, even scalar perturbations on scales $\lambda \ll 15h^{-2}$ Mpc nowadays might well still keep the correlation in question.

c) Reheating after inflation: An instructive example of formation and decay of classical correlations in a quantum system is given by preheating – rapid creation of Bose particles by an oscillating inflaton field in the regime of broad parametric resonance [16]. Here, the practical implementation of the idea [3] that neglection, for each mode, of an exponentially decaying part as compared to the exponentially growing part, is sufficient for effective quantum-to-classical transition has led to the solution of the corresponding classical inhomogeneous wave equation with stochastic initial conditions [17] as a way to go beyond the Hartree-Fock approximation (or its variants like the $\frac{1}{N}$ expansion). In the Hartree-Fock approximation, numerical calculations show the formation of a fixed phase for a given \mathbf{k} mode when the number of created particles $\langle n_{\mathbf{k}} \rangle$ becomes large [18] (this is reflected in particular by the fact that $\langle \phi_{\mathbf{k}}^2 \rangle$ oscillates much more rapidly than the external inflaton field), and no decay of this correlation is seen at late times. On the other hand, higher loop effects like rescattering of created particles which are automatically taken into account in the stochastic numerical simulations finally lead to complete loss of the correlation after a sufficiently large number of oscillations [19].

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